

Why and How the Subject of Mathematics Is So Difficult for Some Children: Study Review and Experience Sharing on Mathematics Disorder

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The present paper gives an account of the different terminologies and definitions of mathematics disorder. In order to answer the question on why and how mathematics is so difficult for some children, related studies are reviewed and the author's clinical experience in working with children having mathematics disorder is shared. The review mainly includes studies on how four domain-specific abilities (i.e., arithmetic procedural skills, number-fact retrieval, place-value concept, and number sense) and two domain-general processing abilities (i.e., working memory and processing speed) may account for children's mathematics learning disabilities. In experience sharing, children's errors made in mathematic tests are analyzed to illustrate their difficulties. Teachers are recommended to pay attention to children's error patterns in low mathematic achievement so as to identify their learning disabilities in mathematics as early as possible.

Key words: mathematics disorder, domain-specific abilities, domain-general abilities

Today, in modern and civilized societies, reading numbers, comparing their magnitudes and doing simple calculations are an essential part of our daily activities. Mathematics learning is not just a school subject; it is also important for us to understand the world around us. Mathematics learning is natural and easy to some people while it can generate failure and create anxiety in others (Chinn, 2004).

The cognitive abilities involved in mathematics learning are complex and varied. The involved cognitive abilities include decoding symbols, logical reasoning, memory, language and visual perceptual skills. In their book “Children Doing Mathematics,” Nunes and Bryant (1996) mention that in order to be numerate, children need to be logical and need to learn conventional systems (i.e., mathematical systems of representation, e.g., numeration system). In addition, children need to use their mathematical thinking meaningfully and appropriately in situations so as to be numerate. All these suggest that mathematics learning is complex, involving different cognitive processes and skills and deficits in any one or more aspects may lead to learning difficulties in mathematics.

Mathematics Learning Disabilities: Definitions and Characteristics

Different terminologies have been used by different psychologists and educational experts in referring to the condition of specific learning difficulty in mathematics. In the *Diagnostic and Statistical Manual of Mental Disorders* (American Psychiatric Association, 2000), the term “Mathematics Disorder” is used. It refers to mathematics ability that is substantially below that is expected for the person’s chronological age, measured intelligence and age appropriate education. In the *International Statistical Classification of Diseases and Related Health Problem* (World Health Organization, 1992), the term used is “Specific Disorder of Arithmetical Skills” which refers to a specific impairment in arithmetical skills that is not solely explicable on the basis of general mental retardation or of inadequate schooling. Meanwhile, the term

“Arithmetical Difficulties” refers to those arithmetical difficulties associated with a reading or spelling disorder. The term “Dyscalculia” is used by Department for Education and Skills (DfES) of United Kingdom and with the notes (in italics) added by Chinn (2003), it is defined as “a perseverant condition that affects the ability to acquire mathematical skills despite appropriate instruction.” (p. 8). Moreover, dyscalculic learners are described as having “difficulty understanding simple number concepts (*such as place value and use of the four operations*), lack an intuitive grasp of numbers (*including the value of numbers and understanding and using the inter-relationship of numbers*), and have problems learning, retrieving and using quickly number facts (*for example multiplication tables*) and procedures (*for example long division*). Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence (*and have no way of knowing or checking that the answer is correct*.” (p. 8).

What is common among these definitions is the presence of perseverant difficulties in learning mathematics despite normal intelligence and appropriate education. They are, however, different in emphases, from more general “mathematics skills” which include abstract skills like algebra, trigonometry, geometry and calculus, to more narrow “arithmetic skills” which involve basic computational skills of the four operations. The definitions are also different in their target inclusion of persons with mathematics learning difficulties with or without reading disabilities versus a specific group of people with specific difficulties in arithmetic skills and no reading disabilities. In the present paper, the focus of discussion is on early primary school children’s arithmetic difficulties, i.e., difficulties in arithmetic computation skills. Children’s mastery of basic arithmetic skills is important for them to build a strong foundation for learning future mathematics concepts. In Hong Kong, arithmetic constitutes a large part of the mathematics curriculum of early primary school years (Curriculum Development Council, 2000). Understanding the cognitive deficits underlying arithmetic learning difficulties will certainly facilitate early identification of children with mathematics disorder.

Cognitive Deficits of Mathematic Disabilities

In the above definition on “dyscalculia” (Chinn, 2003), the learner’s difficulties were described as related to their deficits in conceptual understanding of place value, number sense, fact retrieval and procedural skills. These four basic number skills and knowledge could also be termed as domain-specific abilities which, together with the two domain-general abilities (i.e., working memory and processing speed), were found to be associated with mathematics disabilities in previous research studies. The present paper aims to review related studies in this area. Local examples from the author’s clinical work are used to illustrate the corresponding cognitive deficit in the mathematics performance of children with mathematics disorder.

Arithmetic Procedural skills

A deficit in these skills typically means difficulties in executing arithmetical procedures (e.g., carrying or trading in complex addition), or in executing counting procedures to solve simple addition problems (Geary, 1996). For multi-step arithmetic problems, the procedural errors would include the misalignment of numbers while writing down partial answers, or errors while carrying or borrowing from one column to the next (Russell & Ginsburg, 1984). Children with mathematics disabilities (MD), in comparison with typically-achieving controls, tend to use less sophisticated strategies and commit more errors in solving addition problems (Geary, Hamson, & Hoard, 2000; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Ostad, 1997). However, many of these children improve by the middle of elementary school years (Geary, 2000; Torbeyns, Verschaffel, & Ghesquiere, 2004). Thus, their error-prone use of immature procedures seems to represent a developmental delay instead of a long-term cognitive deficit. Such delay seems to be domain specific and does not characterize children with other academic-related learning deficits. For example, children with only reading disability (RD) do not seem to differ from typically-achieving children in their strategies for solving simple addition problems, in the accuracy of

strategy use (Geary & Hoard, 2002), or in multi-step calculation (Reikeras, 2006).

It is believed that analysing children's errors in mathematics performance can facilitate our understanding of their learning difficulties, and specifically, procedures used by them. With reference to the error classification of Engelhardt (1977), the author has indeed witnessed the following common procedural errors:

1. Carrying error is when the child does addition and forgets about the carrying or mistakes the need of carrying.

For example:

| | |
|-------------|--------------|
| 64 | 209 |
| <u>+ 78</u> | <u>+ 481</u> |
| 132 | 790 |

2. Borrowing error results when the child does subtraction and forgets about borrowing or mistakes the need of borrowing.

For example:

| | |
|-------------|--------------|
| 84 | 703 |
| <u>- 47</u> | <u>- 226</u> |
| 47 | 587 |

3. Inappropriate inversion occurs often when a digit of smaller value is subtracted from a digit of greater value disregarding their positions.

For example:

| | |
|----------------|-----------------|
| 84 | 500 |
| <u>- 47</u> | <u>- 207</u> |
| 43 (7 - 4 = 3) | 307 (7 - 0 = 7) |

4. Incorrect operation refers to the situation when the child performs an operation other than the appropriate one. Examples of this type of error are: $26 - 7 = 33$; $24 \div 8 = 192$.
5. Defective algorithm refers to the execution of a systematic (but erroneous) procedure. There may be omission of critical steps in computations. Examples are: $33 \times 20 = 63$ ($0 \times 3 = 3$; $2 \times 3 = 6$).

Figure 1: Division by a 9-year-old child who has completed P. 4 study

$$\begin{array}{r}
 20 \overline{)820} \\
 \underline{120} \\
 700 \\
 \underline{100} \\
 600 \\
 \underline{100} \\
 500 \\
 \underline{100} \\
 400 \\
 \underline{100} \\
 300 \\
 \underline{100} \\
 200
 \end{array}$$

Theoretically, his algorithm is in the right direction. However, the procedures are so clumsy that he got “lost” and did not know how to deal with all the quotients or answers obtained.

Number-Fact Retrieval

A deficit in number-fact retrieval means difficulties in accessing arithmetic facts from long-term memory (Geary, 2004). One reason why children cannot retrieve a number fact such as “ $5 + 2 = 7$ ” may be interference by other mental operations. For example, in doing a simple addition task, children with MD may favour the “counting-all” strategy which refers to counting from 1, 2, 3...instead of counting up or on from the highest number: 5, 6, 7. The counting procedure may disrupt the association between the problem (i.e., “ $5 + 2$ ”) and answer (i.e., “7”) in long-term memory. A second reason may have to do with difficulties in inhibiting the retrieval of irrelevant associations. For example, children with learning disorders more often incorrectly answer the question “ $5 + 2 = ?$ ” with “6” or “3,” which are numbers that follow the addends in the counting string, than their typically-achieving peers (Geary, Hamson, & Hoard, 2000). Indeed, deficits in number-fact mastery seem rather persistent and quite independent of reading and language abilities (Jordan, Hanich, & Kaplan, 2003a). Number-fact retrieval deficits increasingly emerge as a central characteristic of mathematics disabilities, at least with respect to addition and subtraction operations (Geary, 2004; Gersten, Jordan, & Flojo, 2005; Jordan, Hanich, & Kaplan, 2003b; Ostad, 1998; Robinson, Menchetti, & Torgesen, 2002).

In clinical practice, it was quite often observed that early primary school children had to use finger counting in doing simple addition or subtraction operations. For the MD children, their frequent use of “counting-all” strategy as mentioned above was commonly noted as they could not retrieve the number combination automatically. In the following example, a primary two student wrongly recalled the number fact “ $4 + 8 = 11$ ”. Then, for the tens column of “ $6 + 7$ ”, he/she had to make tally to do the counting and came to the answer 13 but forgot to add the one carried from the units column. This example illustrates the inefficiency and error proneness of children with inadequate number fact retrieval ability.

Figure 2: Use of tally in doing addition by a P. 2 student

$$\begin{array}{r} 64 \\ + 78 \\ \hline 131 \end{array}$$

MD children may also have difficulties reciting the multiplication table or retrieving the multiplication facts. The following are examples collected by the author during clinical practice:

Figure 3: Performance by a 8 year-old child in a one-minute multiplication task

$$\begin{array}{l} 9 \times 3 = \underline{X} \\ 5 \times 5 = \underline{X} \\ 4 \times 8 = \underline{X} \\ 3 \times 2 = \underline{X} \\ 6 \times 4 = \underline{X} \\ 3 \times 7 = \underline{31} \end{array}$$

Figure 4: Performance by a P. 3 student with mistake on multiplication fact retrieval

$$\begin{array}{r} 105 \\ \times 47 \\ \hline 742 \end{array}$$

Conceptual Understanding of Place Value

A key insight about numbers is that the value of a digit depends on its place in a group of digits (Chinn & Ashcroft, 1999). For example, while most 2nd grade children had some trouble with positional knowledge and digit correspondence, typically-achieving children outperformed children with reading and/or mathematical difficulties (Hanich, Jordan, Kaplan, & Dick, 2001). A reliable connection between place-value understanding and addition and subtraction skills among Chinese children has also been documented. Moreover, training in place-value concept effectively improved the Chinese children's place-value understanding as well as addition skills (Ho & Cheng, 1997).

Children's difficulties in place value can be reflected in many different ways. Here are some examples:

Figure 5: Constructing a 4-digit integer of the smallest value by a P. 5 student

請用 6,0,8,2 這些數字組成為一個最小數值的四位數 2680 ✗
 Construct a 4-digit integer of the smallest value using the numbers 6,0,8,2

Figure 6: Transcoding from Chinese into Arabic numerals by a P. 4 student with dyslexia

用阿拉伯數字寫出下列各數：

Express the following in Arabic numerals

i) 四千二百三十六 40002836

Four thousand two hundred and thirty-six

ii) 一萬六千零十七 10000607

Sixteen thousand and seventeen

iii) 七千零六 700006

Seven thousand and six

Figure 7: Counting the blocks with poor understanding of the pictorial representation by a P. 5 student

下圖中有積木多少粒？ 48012
How many blocks are there in the following figure?

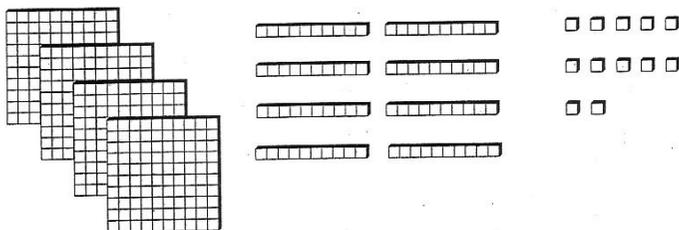
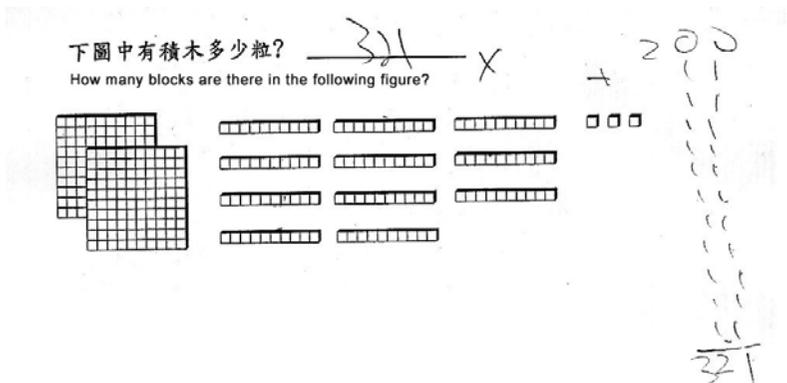


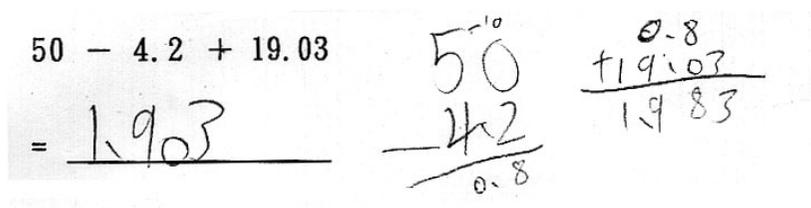
Figure 8: Counting the blocks with partial understanding of pictorial representation by a P. 4 student



Firstly, he understood the representation of two hundreds but then mistakenly counted the tens bars as consisting of 11 blocks. Failing to apply the multiplication rule, he added all the eleven bars of 11 blocks to two hundreds and came up to the answer of 321, ignoring the 3 separate blocks.

Children’s conceptual understanding of place value is even more challenged when decimal is involved. Below is an example:

Figure 9: Calculations with decimal by a P. 5 student



Firstly, the child misaligned the digit 4 (units place) to the digit 5 (tens place) and then did the subtraction. The answer of the subtraction was then added to 19.03. This time, he aligned the digits of different place values correctly. However, in putting the decimal to the answer, he counted the number of digits after the decimal in both addends (the rule for multiplication).

Number Sense

Number sense was defined by Malofeeva, Day, Saco, Young, & Ciancio (2004) as “an understanding of what numbers mean and of numerical relationships” (p. 658). According to Gersten and Chard (1999), the concept of number sense was described as “an analog as important to mathematics learning as phonemic awareness has been to the reading research field.” (p. 18). It plays a significant role in children’s general ability to feel at ease with numbers and make confident progress in the number-work in early school years. It seems to predict children’s general ability to feel at ease with numbers and to make confident progress in number-work in early school years. Often acquired informally, it is basic and important in early formal arithmetic learning. Kindergarteners’ number sense performance and growth together accounted for 66% of the variance in first grade mathematics achievement in a study by Jordan, Kaplan, Locuniak, and Ramineni (2007). Longitudinal studies suggested that screening early number-sense development could help identify children who might develop mathematics difficulties or disabilities later (Gersten, Jordan, & Flojo, 2005; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Kaplan, Olah, & Locuniak, 2006). Because number sense is related to the meaning of numbers stored in long-term memory, Robinson et al. (2002) speculated, weak number sense might contribute to number-fact retrieval deficits in children with MD. One problem with this account is that the construct of number sense might be too general to account for specifics about MD. For instance, at least some elements of number sense seem intact in children with MD-only, as evidenced by their relatively strong performance in problem solving involving numbers and in using counting procedures (Jordan et al., 2003b).

In clinical practice, young children's number sense may sometimes be assessed by asking them to compare two numerals. A child with poor number sense may say that "98" is bigger than "101" as "9" is always more than "1". Or, "6019" is more than "6100" as they may compare the last digit only. Moreover, they may have no idea in deciding whether "22" is closer in magnitude to "14" or to "52"; or they may consider "300" is closer to "500" than to "199".

Working Memory

The mental capacity for temporary processing and storage of information (i.e., working memory) has been a focus in research on mathematical performance (Rosselli, Matute, Pinto, & Ardila, 2006). While many would agree that children with mathematics difficulties seem to have some degree of working memory deficit (e.g., Bull, Johnston, & Roy, 1999; Geary et al., 2004; McLean & Hitch, 1999; Rosselli et al., 2006; Wilson & Swanson, 2001), contribution of specific working memory components to mathematics difficulties remains unclear.

DeStefano and LeFevre (2004) suggested that all three components proposed by Baddeley (Baddeley & Logie, 1999) — i.e., the phonological loop, the visuospatial sketchpad, and the central executive of the working memory system—could play a role in mental arithmetic. Yet, some studies suggested that the relation between working memory and mathematics performance varied as a function of age and ability (Andersson & Lyxell, 2007), with a stronger role for the visuospatial sketchpad in the younger children's mathematics performance (Holmes & Adams, 2006). Other studies found that central executive was the main impairment in children with MD (Andersson & Lyxell, 2007; Bull et al., 1999; McLean & Hitch, 1999; Passolunghi & Siegel, 2004; Wilson & Swanson, 2001). The central executive of working memory seems to be a core deficit for children with MD, whereas the phonological loop and visuospatial sketchpad may contribute more specific math-related cognitive deficits (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Still other studies concluded that children with MD had problems with working memory tasks involving numerical

information and not those involving non-numerical verbal information (Geary, Brown, & Samaranayake, 1991; McLean & Hitch, 1999; Passolunghi & Siegel, 2001; Siegel & Ryan, 1989). In other words, no consistent findings have emerged among mathematics difficulties and the various working-memory components, and further studies are needed.

In local practice, it is common among clinicians to administer the “Digit Span” subtest of the Hong Kong Wechsler Intelligence Scale for Children (HK-WISC), which is also a measure of the working memory to children suspected of having specific learning difficulties. It is found that quite often, children with MD show difficulties when they have to recall digits presented to them orally in backward sequence.

Processing Speed

Children with MD are generally slow in solving arithmetic problems (e.g., Ostad, 2000). Perhaps they simply take more time than their typically-achieving peers to execute all basic numerical processes, or perhaps they favour slower counting strategies (e.g., counting-all) rather than the faster number-fact retrieval strategies (Geary, 1996). With little increase in retrieval speed with age, many MD children lag further and further behind their peers. Since arithmetic ability seems strongly predicted by processing speed (e.g., Bull & Johnston, 1997; Fuchs, Fuchs, Compton, Powell, Seethaler, & Capizzi, 2006), arithmetic difficulties persist perhaps due to persistent processing-speed deficit, which hampers automatic basic arithmetic-facts retrieval (e.g., Geary et al., 2007). Indeed, Jordan and Montani (1997) found that children with MD-only performed worse than the control group on both story and number-fact problems in timed conditions but not in untimed conditions, suggesting domain-general processing-speed deficits. By contrast, children with both mathematics and reading difficulties performed worse than the control group regardless whether the tasks were timed, suggesting more general deficits in problem conceptualization and execution of calculation procedures.

Consistent with the research findings, young children with learning disabilities in mathematics were observed to be slow at work in our

clinical setting. This is closely related to their poor number sense and taking more time in comprehension of the question, use of inefficient counting strategy instead of faster number-fact retrieval strategy, and uncertainties of the answer obtained with time spent in re-checking. However, with time, if no effective intervention is offered to them, it is not uncommon for the older children with MD to just give up items with difficulties such as long divisions, or computations with decimal in tests or examinations. In many instances, these students may “finish” the paper well before the time limit.

Conclusion: What Teachers Can Do

Mathematics can be very difficult for those children with one or more of the above mentioned deficits in the four domain specific abilities (i.e., arithmetic procedural skills, number-fact retrieval, place-value concept, and number sense) and two domain-general processing abilities (i.e., working memory and processing speed). Teachers can observe students’ working process and analyze their error patterns in the assignments or test papers so as to identify their difficulties. Early intervention should follow identification. Teaching of proper procedural knowledge and skills should be beneficial and training of place-value concept was found to be effective in improving children’s addition skills (Ho & Cheng, 1997). For those children with normal intelligence but persistent low achievement in mathematics, referral should be made to the school educational psychologist for formal assessment and diagnosis. As with other types of learning disabilities, early identification and intervention are both important. For mathematics, the issue may be complicated by math anxiety and all the other undesirable psychological side effects such as poor self esteem, if the problem is not managed properly in early primary school years.

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「數學真難呀！」之為何及如何：有關數學障礙的研究評論及臨床經驗分享

陳美賢

摘要

本文首先介紹有關數學障礙的不同學術名稱及定義，再以相關的研究及臨床經驗，幫助讀者了解為甚麼有些孩子認為「數學真難呀！」，進而探討他們的難處何在。研究顯示數學障礙主要與四項數字相關的能力（即運算步驟的掌握、數字組合的檢索能力、位值概念與數字感）及兩項腦部運作能力（即工作記憶與訊息處理速度）有關。至於經驗分享的部分，筆者對孩子在數學測驗中所犯的錯誤作出分析，並展示他們在某項能力上所遇到的困難。最後，筆者提議老師多留意孩子在數學表現上常犯錯誤的模式，務求及早識別他們在數學方面的學習障礙。

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